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OPTIMAL CONTROL OF A SYSTEM WITH INTERMEDIATE CONDITIONS*

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Necessary optimality conditions are derived in the general problem of control with intermediate constraints on the trajectory. It is shown that many well-known results (see/1—11/, for example) are some realization or other of the general Lagrange multiplier rule. Statements, strenghthened in comparison with existing ones, of necessary optimality conditions are derived for two important cases (composite and discontinuous) of the general problem. The importance of certain conditions is verifed by an example.

1. Statement of the problem. Let there be given: an interval I of the real line $R=(-\infty,\infty)$, sets U, W of finite-dimensional real spaces R^r, R^l and a subset T of the cube $I^{m+1}\subset R^{m+1}$. For a function $x\colon I\to R^n$ and a point $\theta=(\theta_0,\ldots,\theta_m)$ from T we set $x(\theta)=(x(\theta_0),\ldots,x(\theta_m))$. We consider the control problem (Problem A)

$$\begin{aligned}
&\Phi_0(x(\theta),\ \omega,\ \theta)\to\min\\
&\Phi_j(x(\theta),w,\theta)\leqslant 0,\ j=1,2,\ldots,p\\
&\Phi_j(x(\theta),w,\theta)=0,\ j=p+1,\ldots,q\\
&x^{\cdot}=f(x,u,w,t);\, x\in R^n,\, u\in U,\, w\in W,\, \theta\in T
\end{aligned}$$

in which we seek the minimum of functional Φ_0 over controls $u(\cdot)$ and parameters w,θ under constraints of inequality, equality and inclusion types.

We make the following assumptions. The scalar functions $\Phi_j(x^0,\ldots,x^m,w,\theta)$ are differentiable with respect to the arguments on an open set containing the Cartesian product $R^{n(m+1)}\times W\times T$; the mapping f from $R^n\times U\times W\times R$ into R^n is continuous and for fixed u,t has continuous partial derivatives f_x, f_w in a neighborhood of $R^n\times W$; the set U is bounded; $q-p<(m+1)(n+1)+l$. A piecewise-continuous function $u\colon R\to U$ is called a control, while the quadruple $x(\cdot), u(\cdot), w, \theta$ made up of the control $u(\cdot)$, the parameters w, θ, and the corresponding continuous piecewise-smooth solution $x(\cdot)$ of the differential equation, satisfying the problem's constraints, is called an admissible process. An admissible process $x(\cdot)$, $u(\cdot)$, w, θ is considered optimal if a number $\delta>0$ can be found such that the inequality $\Phi_0(x(\theta),w,\theta)\leqslant\Phi(x^*(\theta^*),w^*,\theta^*)$ is fulfilled for any admissible process $x^*(\cdot), u^*(\cdot), w^*, \theta^*$ answering to the conditions $|x^*(t)-x(t)|<\delta$, $t\in Q$, $|w^*-w|<\delta$, $|\theta^*-\theta|<\delta$. Here $|z|$ is the Euclidean norm of vector z, Q is the smallest segment containing the coordinates of vectors θ,θ^*. Sufficiently general problems /1—11/ without phase or mixed constraints reduce to the form indicated. Two special cases of Problem A for a composite and a discontinuous control system are considered below in Sects. 4 and 5.

2. Necessary optimality conditions. Theorem 1. Let $x(\cdot)$, $u(\cdot), w, \theta$ be a process admissible on the interval (t_0, t_1), in which the coordinates of vector θ are ordered by growth and are points of continuity of the control. For the process' optimality it is necessary that there exist a vector $\lambda=(\lambda_0,\ldots,\lambda_q)$ and a solution, piecewise-continuous on $[\theta_0,\theta_m]$ of the solution $\psi(t)$ of the adjoint equation

$$\begin{aligned}
&\psi^{\cdot}=-H_x(\psi,x(t),u(t),w,t),\ t\neq\theta_m,\theta_{m-1},\ldots,\theta_0\\
&\psi(\theta_k-)=\psi(\theta_k+)-L_{x^k}(\lambda,\ x(\theta),\ w,\ \theta),\ k=m,\ m-2,\ldots,1
\end{aligned}$$

such that the following conditions are valid:

1) nontriviality, sign-definiteness and complementing nonrigidity for vector λ

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$$|\lambda| > 0;\ \lambda_j \geqslant 0,\ j = 0, 1, \ldots, p$$
$$\lambda_j \Phi_j (x(\theta), w, \theta) = 0,\ j = 1, 2, \ldots, p$$

2) transversality for the trajectory's endpoints

$$\psi(\theta_m) = -L_{x^m}(\lambda, x(\theta), w, \theta),\ \psi(\theta_0) = L_{x^0}(\lambda, x(\theta), w, \theta)$$

3) maximum of the Hamiltonian for the control

$$H(\psi(t), x(t), u(t), w, t) = \max_{v \in U} H(\psi(t), x(t), v, w, t),\ \theta_0 \leqslant t \leqslant \theta_m$$

4) optimality of parameter w

$$\left[L_w(\lambda, x(\theta), w, \theta) - \int_{\theta_0}^{\theta_m} H_w(\psi(t), x(t), u(t), w, t)\, dt\right]' \delta w \geqslant 0,\ \delta w \in K(w, W)$$

5) optimality of parameter θ

$$L_\theta(\lambda, x(\theta), w, \theta)' \delta\theta \geqslant 0,\ \delta\theta \in K(\theta, T)$$

Here

$$H(\psi, x, u, w, t) = \psi' f(x, u, w, t)$$
$$L(\lambda, x^0, \ldots, x^m, w, \theta) = \sum_{j=0}^{q} \lambda_j \Phi_j(x^0, \ldots, x^m, w, \theta)$$

$K(w, W)$ is the conic approximation of set W at point w, which, be definition, has the following property: for any finite collection of vectors $\delta w^1, \ldots, \delta w^s$ from $K(w, W)$ and for an arbitrary $\varepsilon > 0$ we can find a function $o: R^s \to R^l$ such that

$$w + \sum_{i=1}^{s} \xi_i \delta w^i + o(\xi) \in W,\quad \xi = (\xi_1, \ldots, \xi_s) \geqslant 0,\ |\xi| < \varepsilon$$
$$|o(\xi)|/|\xi| \to 0,\ |\xi| \to 0$$

The conic approximation $K(\theta, T)$ is defined analogously. The vectors participating in the operations are reckoned to be column vectors; the prime denotes transposition. By convention the symbol $x'y$ is the scalar product of vectors x and y. The derivative L_θ in condition 5) is taken relative to the differential equation system by the formula

$$L_\theta(\lambda, x, (\theta), w, \theta) = (L_{x^0}(\lambda, x(\theta), w, \theta)' f|_{\theta_0} + L_{\theta_0}(\lambda, x(\theta), w, \theta), \ldots, L_{x^m}(\lambda, x(\theta), w, \theta)' f|_{\theta_m} + L_{\theta_m}(\lambda, x(\theta), w, \theta))$$

Notes. 1°. For interior points $\theta \in T$ whose coordinates are the control's points of discontinuity, instead of 5) there hold the conditions

$$\pm [L_{x^k}(\lambda, x(\theta), w, \theta)' f|_{\theta_k \pm} + L_{\theta_k}(\lambda, x, (\theta), w, \theta)] \geqslant 0,\ k = 0, 1, \ldots, m$$

where $f|_{\theta_k -}, f|_{\theta_k +}$ are one-sided limits of the function $t \to f(x(t), u(t), w, t)$ at point $t = \theta_k$.

2°. If among the coordinates of θ there are equal ones, say, $\theta_{i-1} < \theta_i = \theta_{i+1} = \ldots = \theta_j < \theta_{j+1}$, then from the jump condition for the adjoint function follows

$$\psi(\theta_i -) = \psi(\theta_j +) - \sum_{k=i}^{j} L_{x^k}(\lambda, x(\theta), w, \theta)$$

The proof of Theorem 1 consists in constructing the cones of variations of the optimal trajectory, with a subsequent application of the convex set separability theorem /12/, or in the reduction of Problem A to a special finite-dimensional mathematical programming problem in which the parameters of the optimal process' variations /13/ are the unknowns. Although it requires the proof that the Lagrange multipliers are independent of the variation's parameters, the second method is shorter.

3. Discussion of the results. Let us compare the statement of Problem A and the necessary conditions obtained with those known in the literature. The case when the data of Problem A are independent of the parameters, the intermediate controls are absent and the set $T \subset R^2$ is described by the inequality $\theta_0 < \theta_1$ has been worked on repeatedly (see /11,13/, for instance). The corresponding statements of Theorem 1 coincide with the ones known; in this connection condition 4) is trivial, while from 5) follows $L_\theta = 0$ because set T is open. A variant was suggested in /6/ of a variational problem with time discontinuities, i.e., the control process is examined on several pairwise-nonintersecting or abutting each other intervals $\theta_{2k} \leqslant t \leqslant \theta_{2k+1}$, $k = 0, 1, \ldots, h$. The values of the trajectory at the intervals' endpoints are related by compatible equality constraints. This variant reduces to Problem A if we set $m = 2h + 2$ and specify set T by the inequalities $\theta_0 \leqslant \theta_1 \leqslant \ldots \leqslant \theta_m$. In /6/ it was proposed that the optimal values of parameter θ be found from the equation $L_\theta = 0$. By condition 5) the latter is true for the interior points of set T. In the general case this condition does not obtain for the boundary points. Control problems for step systems /2,5/, with discontinuous constraints on the control /4/ or with separated discontinuities of the phase trajectories /8/, are special cases of Problem A ; the corresponding results ensue from Theorem 1. A stationarity condition for the Hamiltonian along the optimal process has been obtained in /7/ for Problem A under the assumptions that set T is open and the system is controllable. As seen from the statement of Theorem 1 a stronger condition 3) is true, and without the additional assumptions mentioned above.

4. Composite systems. Following /2/, a control system is called composite if it is described on different time intervals by different differential equations and by certain end connections of the trajectories' junctions. Let us analyze in detail one composite control system problem (Problem B)

$$\Phi_0 (x^1(\theta_{01}), \ldots, x^N(\theta_{0N});\ x^1(\theta_{11}), \ldots, x^N(\theta_{1N});\ w^1, \ldots, w^N;$$
$$\theta_{01}, \ldots, \theta_{0N};\ \theta_{11}, \ldots, \theta_{1N}) \to \min$$
$$\Phi_j^i (x^{i+1}(\theta_{0,i+1}), x^i(\theta_{1i}), w^i, w^{i+1}, \theta_{0,i+1}, \theta_{1i}) \leqslant 0,\ j = 1, 2, \ldots, p_i$$
$$\Phi_j^i (x^{i+1}(\theta_{0,i+1}), x^i(\theta_{1i}), w^i, w^{i+1}, \theta_{0,i+1}, \theta_{1i}) = 0,\ j = p_i + 1, \ldots, q_i,\ i = 0, 1, \ldots, N$$
$$dx^i/dt = f^i(x^i, u^i, w^i, t),\ \theta_{0i} \leqslant t \leqslant \theta_{1i}$$
$$x^i \in R^{n_i},\ u^i \in U^i \subset R^{r_i},\ w^i \in W^i \in R^{l_i},\ \theta_{0i} < \theta_{1i}$$
$$i = 1, 2, \ldots, N$$

where Φ_0, Φ_j^i are smooth functions of the arguments

$$y^k \equiv x^k(\theta_{0k}),\ z^k \equiv x^k(\theta_{1k}),\ w^k,\ \theta_{0k},\ \theta_{1k}, \quad k = 0, 1, \ldots, N$$

independent of z^0, w^0, θ_{10} when $i = 0$ and of $y^{N+1}, w^{N+1}, \theta_{0,N+1}$ when $i = N$, and the f^i are functions with the same analytic properties as f. Problem B reduces to Problem A if we set

$$R^n = R^{n_1} \times \ldots \times R^{n_N},\ U = U^1 \times \ldots \times U^N$$
$$W = W^1 \times \ldots \times W^N$$
$$\theta = (\theta_{01}, \ldots, \theta_{0N}, \theta_{11}, \ldots, \theta_{1N})$$

and specify the set $T \subset R^{2N}$ by the inequalities $\theta_{0i} < \theta_{1i}$, $i = 1, 2, \ldots, N$. Let us write out the necessary optimality conditions for Problem B. Without loss of generality the controls $u^i(\cdot)$ can be taken to be continuous at the ends of the segments $[\theta_{0i}, \theta_{1i}]$; therefore, we obtain the following result as a corollary to Theorem 1.

Theorem 2. For the optimality of the admissible $x^i(\cdot), u^i(\cdot), w^i, \theta_{0i}, \theta_{1i}$, $i = 1, 2, \ldots, N$ in Problem B it is necessary that there exist a nontrivial collection of numbers λ_0, λ_j^i, $j = 1, 2, \ldots, q_i$, $i = 0, 1, \ldots, N$ and the solutions $\psi^i(t)$, $i = 1, 2, \ldots, N$, continuous on $[\theta_{0i}, \theta_{1i}]$, of the adjoint equations

$$d\psi^i/dt = -H_{x^i}^i(\psi^i, x^i(t), u^i(t), w^i, t),\ \theta_{0i} < t < \theta_{1i}$$
$$i = 1, 2, \ldots, N$$

such that the following relations are valid:

$$1)\ \lambda_0 \geqslant 0,\quad \lambda_j^i \geqslant 0,\quad j = 1,2,\ldots, q_i,\ i = 0,1,\ldots, N$$
$$\lambda_j^i \Phi_j^i(\cdot) = 0,\quad j = 1,2,\ldots, q_i,\ i = 0,1,\ldots, N$$
$$2)\ \psi^i(\theta_{0i}) = L_{y^i}(\cdot),\ \psi^i(\theta_{1i}) = -L_{z^i}(\cdot),\ i = 1,2,\ldots, N$$
$$3)\ H^i(\psi^i(t), x^i(t), u^i(t), w^i, t) = \max_{v\in U^i} H^i(\psi^i(t), x^i(t), v, w^i, t)$$
$$\theta_{0i} \leqslant t < \theta_{1i},\quad i = 1, 2, \ldots, N$$
$$4)\ \left[L_{w^i}(\cdot) - \int_{\theta_{0i}}^{\theta_{1i}} H_{w^i}^i(\psi^i(t), x^i(t), u^i(t), w^i, t)\,dt\right]'\delta w \geqslant 0$$
$$\delta w \in K(w^i, W^i),\quad i = 1, 2, \ldots, N$$
$$5)\ L_{y^i}(\cdot)' f^i(x^i(\theta_{0i}),\ u^i(\theta_{0i}),\ w^i,\ \theta_{0i}) + L_{\theta_{0i}}(\cdot) = 0$$
$$L_{z^i}(\cdot)' f^i(x^i(\theta_{1i}),\ u^i(\theta_{1i}),\ w^i,\ \theta_{1i}) + L_{\theta_{1i}}(\cdot) = 0$$
$$i = 1, 2, \ldots, N$$

Here

$$H^i(\psi^i, x^i, u^i, w^i, t) = (\psi^i)' f(x^i, u^i, w^i, t)$$
$$L(\cdot) = \lambda_0\Phi_0(\cdot) + \sum_{i=0}^{N}\sum_{j=1}^{q_i} \lambda_j^i \Phi_j^i(\cdot)$$

The symbol $(\cdot)$ denotes the collection of arguments of the corresponding functions and of their derivatives on the optimal process.

Problems with intermediate limitations of inequalities type at a nonfixed instant /1,2,4, 5/ (also /7/, pp.123, 127 (of the Russian translation)) or with an inequality type condition at a fixed instant (/10/, p.174) are covered in the statement of Problem B; the corresponding results follow from Theorem 1. We note that the theorem's assertions are valid without the additional assumptions of nondegeneracy and controllability of the systems, adopted in /2,7/.

5. Discontinuous systems. By a discontinuous system we mean a system of ordinary differential equations with a piecewise-continuous right-hand side. Necessary optimality conditions for discontinuous control systems are very well known /14—18/ when the optimal trajectory intersects the surface of discontinuity without one-sided tangencies. Cases of tangency were studied /19,20/ under rather strong assumptions on the problem's data. We show below that a part of the assumptions can be eliminated if the discontinuous control system is reduced /1/ to a problem with intermediate controls and then Theorem 1 is used. Consider the discontinuous control problem (Problem C)

$$\Phi_0(x(t_0), x(t_1), t_0, t_1) \to \min$$
$$\Phi_j(x(t_0), x(t_1), t_0, t_1) \leqslant 0,\ j = 1, 2,\ldots, p$$
$$\Phi_j(x(t_0), x(t_1), t_0, t_1) = 0,\ j = p+1,\ldots, q$$
$$x^\cdot = \begin{cases} f^-(x, u, t), & g(x,t) < 0 \\ f^+(x, u, t), & g(x,t) > 0 \end{cases}$$
$$x \in R^n,\ u \in U \subset R^r,\ t_0 < t_1$$

Here $\Phi_j(x^0, x^1, t_0, t_1)$ and $g(x, t)$ are smooth scalar functions defined, respectively, on $R^n \times R^n \times R \times R$ and $R^n \times R$, f^-, f^+ are continuous mappings from $R^n \times U \times R$ into R^n, with continuous partial derivatives f_x^-, f_x^+, $q - p < 2n + 2$. Let the problem's solution comprise of: a pair of numbers $t_0 < t_1$, a control $u : [t_0, t_1] \to U$ from the class of piecewise-continuous functions, and a continuous piecewise-smooth function $x : [t_0, t_1] \to R^n$ satisfying the differential equation in the sense of /21/. Assume further that the equation $g(x(t), t) = 0$ has a single root $t = \tau$ on the interval (t_0, t_1). Following the scheme outlined, we set up the auxiliary problem (Problem D)

$$\Phi_0(y(\theta_0), z(\theta_2), \theta_0, \theta_2) \to \min$$
$$\Phi_j(y(\theta_0), z(\theta_2), \theta_0, \theta_2) \leqslant 0,\ j = 1,2,\ldots, p$$
$$\Phi_j(y(\theta_0), z(\theta_2), \theta_0, \theta_2) = 0,\ j = p+1,\ldots, q$$
$$g(y(\theta_1), \theta_1) = 0,\ y(\theta_1) - z(\theta_1) = 0$$
$$y^\cdot = f^1(y, u, t),\ z^\cdot = f^2(z, u, t),\quad y \in R^n,\ z \in R^n,\ u \in U,\ \theta_0 < \theta_1 < \theta_2$$

The functions f^1, f^2 are defined thus on the set $R^n \times U \times R$. The set of solutions of the inequalities $g \leqslant 0$, $g \geqslant 0$ are denoted G^-, G^+, respectively. Obviously, $G^-, G^+ \subset R^{n+1}$. We set $f^1 = f^2 = f^-$ if for $t_0 \leqslant t \leqslant t_1$ the graph of function $x(t)$ lies in G^- and $f^1 = f^-$, $f^2 = f^+$ if the graph successively intersects the kernels of G^-, G^+. These functions are specified analogously in the remaining cases. Problems C and D are said to be compatible if an optimal process $y^*(\cdot), z^*(\cdot), u^*(\cdot), \theta_0^*, \theta_1^*, \theta_2^*$ of Problem D exists generating an admissible process of Problem C: $x = y^*(t)$, $u = u^*(t)$, $\theta_0^* \leqslant t \leqslant \theta_1^*$; $x = z^*(t)$, $u = u^*(t)$, $\theta_1^* \leqslant t \leqslant \theta_2^*$. We see that under the condition of compatibility of Problems C and D the process $y(\cdot), z(\cdot), u(\cdot)$, $\theta_0 = t_0$, $\theta_1 = \tau$, $\theta_2 = t_1$ corresponding to the optimal control $u(\cdot)$ and to the initial condition $y(\tau) = z(\tau) = x(\tau)$ satisfies all the conditions of Problem D and is optimal. Let us write out the necessary optimality conditions for it (Theorem 1). Since these conditions are simultaneously necessary for the optimality of the process $x(\cdot), u(\cdot), t_0, t_1$ in Problem C,the final conclusion can be given as:

Theorem 3. Let Problems C and D be compatible. If the process $x(\cdot), u(\cdot), t_0, t_1$ of Problem C is optimal and along it the equation $g(x(t), t) = 0$ has a single root $t = \tau$ in the interval (t_0, t_1) then there exist a vector $\lambda = (\lambda_0, \ldots, \lambda_q)$, a number μ, and a solution $\psi(t)$, continuous for $t \in [t_0, t_1]$, $t \neq \tau$, of the adjoint differential equation

$$\psi^{\cdot} = -H_x(\psi, x(t), u(t), t),\ t \neq \tau, \quad \psi(\tau -) = \psi(\tau +) + \mu g_x(x(\tau), \tau)$$

such that

$$1)\ |\lambda| + |\mu| > 0;\ \lambda_j \geqslant 0,\ j = 0, 1, \ldots, p$$
$$\lambda_j \Phi_j(x(t_0), x(t_1), t_0, t_1) = 0,\ j = 1, 2, \ldots, p$$
$$2)\ \psi(t_1) = -L_{x^1}(\lambda, x(t_0), x(t_1), t_0, t_1)$$
$$\psi(t_0) = L_{x^0}(\lambda, x(t_0), x(t_1), t_0, t_1)$$
$$3)\ H(\psi(t), x(t), u(t), t) = \max_{v \in U} H(\psi(t), x(t), v, t),\ t_0 \leqslant t \leqslant t_1$$
$$4)\ L_{x^0}(\lambda, x(t_0), x(t_1), t_0, t_1)' x^{\cdot}(t_0 +) + L_{t_0}(\lambda, x(t_0), x(t_1), t_0, t_1) = 0$$
$$L_{x^1}(\lambda, x(t_0), x(t_1), t_0, t_1)' x^{\cdot}(t_1 -) + L_{t_1}(\lambda, x(t_0), x(t_1), t_0, t_1) = 0$$
$$\psi(\tau +)' [x^{\cdot}(\tau -) - x^{\cdot}(\tau +)] + \mu [g_x(x(\tau), \tau)' x^{\cdot}(\tau -) + g_t(x(\tau), \tau)] = 0$$

Here

$$H(\psi, x, u, t) = \begin{cases} \psi' f^-(x, u, t), & g(x, t) < 0 \\ \psi' f^+(x, u, t), & g(x, t) > 0 \end{cases}$$

$$L(\lambda, x^0, x^1, t_0, t_1) = \sum_{j=0}^{q} \lambda_j \Phi_j(x^0, x^1, t_0, t_1)$$

Theorem 3 generalizes the results in /14—17/ in two directions: to more general discontinuous problems with consistent constraints on the trajectory's endpoints and to the case of tangency, characterized by the equalities $g(x(\tau), \tau) = 0$, $g_x(x(\tau), \tau)' x^{\cdot}(\tau -) + g_t(x(\tau), \tau) = 0$. In the majority of investigations /12,14,16—18/ the tangency case is excluded from analysis. In /19,20/ it is examined under rather rigid assumptions on the functions f^-, f^+. As we see from the statement of Theorem 3, the anomality of the tangency case shows up in the fact that the last condition in 4) determined not the multiplier μ but yields the additional relation

$$5)\ \psi(\tau +)' [x^{\cdot}(\tau -) - x^{\cdot}(\tau +)] = 0$$

on the adjoint variables and, through them, on the vector λ. The latter circumstance proves to be very essential.

Example. Let the conditions of Problem C be

$$\varepsilon x_1(t_1) + x_2(t_1)/\varepsilon \to \min;\ t_0 = 0,\ t_1 - 2 = 0$$
$$x_1(t_0) = x_2(t_0) = 0;\ x_1^{\cdot} = u_1,\ x_2^{\cdot} = u_2,\ g = -x_2 - (x_1 - 1)^2 < 0$$
$$x_1^{\cdot} = 0,\ x_2^{\cdot} = -1,\ g > 0;\ 0 \leqslant u_1, u_2 \leqslant 1$$

For a small $\varepsilon > 0$ the control $u_1(t) = 1$, $u_2(t) = 0$, $0 \leqslant t \leqslant 2$, is optimal; the corresponding trajectory $x_1(t) = t$, $x_2(t) = 0$, $0 \leqslant t \leqslant 1$; $x_1(t) = 0$, $x_2(t) = 1 - t$, $1 \leqslant t \leqslant 2$ is tangent to the line of discontinuity $g = 0$ at the instant $\tau = 1$. In this case the hypotheses of Theorem 3 are fulfilled and the necessary optimality conditions are satisfied for $\lambda_0 = 0$, $\mu < 0$. The equality $\lambda_0 = 0$ turns out to be a consequence of requirement 5). We note that conditions 5) and 3) are not fulfilled when $\lambda_0 = 1$. Hence it follows that the results in /15/ are, in general, inapplicable to the tangency case. The example here shows as well that in discontinuous control systems a direct analog of the maximum principle (relations 1)—4) of Theorem 3 without the last equality in

4)) is not valid in the general case even with discontinuous adjoint functions. From it follows, however, that piecewise linearity of the discontinuous system with respect to the phase variable does not guarantee the convexity of the attainability set. By direct calculations it can be verified that at instant $t=2$ the attainability set is the union of a square and of an isolated point on the plane.

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